ing; h, height of a projection (depth of a depression); d_{in}, inside diameter of pipe along a smooth section; Nu, Re, Pr, St, Nusselt, Reynolds, Prandtl, and Stanton numbers, respectively. Indices: t, turbulent number (region of turbulent transfer); m, region of molecular transfer; w, wall; s, outer boundary of a region; f, average value; vort, vortex region; sm, region of "smooth" flow (smooth pipe); in, inside; pr, profiled pipe.

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ANALYSIS OF LONGITUDINAL VELOCITY FLUCTUATIONS ON A PLATE

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UDC 532.517.4

It is proposed to use the equation of the second moments with a mixing path length determined on the basis of experimental data for the longitudinal velocity fluctuations in a boundary layer.

In describing a number of hydrodynamics and heat-transfer problems, not only the averaged but also the fluctuation characteristics of the flow must be known. As an illustration, turbulent transfer processes in apparatus of chemical technology, high-temperature energetics, space and laser engineering can be cited. The heat flux in such apparatus depends not only on the turbulent transfer coefficients and the mean flow parameters, but also on the fluctuation structure of the flow, since it exerts substantial influence on the rate of physicochemical transformation and, consequently, on the heat and mass transfer.

Many paper [1-4], say, are devoted to the experimental investigation of fluctuating turbulent flow structure. Mainly problems of closing the turbulent transfer equations have been worked out theoretically [5-9]. The description and analysis of singularities in the velocity and temperature fluctuation distributions are limited.

An attempt is made in this paper to compute the longitudinal velocity fluctuation profile in the boundary layer on a plate around which a gradient-free gas flows. The problem of determining the average velocity has been studied sufficiently well for this case. The velocity profile is determined from the equations

$$\frac{\partial \rho \bar{u}}{\partial x} + \frac{\partial \rho \bar{v}}{\partial y} = 0; \tag{1}$$

Moscow Power Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 5, pp. 723-727, November, 1985. Original article submitted October 29, 1984.

0022-0841/85/4905-1258$09.50

$$\rho\bar{u}\frac{\partial\bar{u}}{\partial x}+\rho\bar{v}\frac{\partial\bar{u}}{\partial y}=\frac{\partial}{\partial y}\left[\rho(\nu+\nu_T)\frac{\partial\bar{u}}{\partial y}\right] \tag{1}$$

with the boundary conditions

$$y=0\quad \bar{u}=\bar{v}=0;\ y\to\infty\quad \bar{u}\to\bar{u}_\infty .$$

The Prandtl hypothesis about the mixing path is used to close this system. The scale of turbulence was computed by the Simpson rule with the Van Driest correction that takes account of the damping effect of the wall [4]:

$$\nu_T=L^2\frac{\partial\bar{u}}{\partial y}; \tag{2}$$

$$L=\varkappa\left[1-\exp\left(-\frac{4y}{\delta}\right)\right]\left[1-\exp\left(-\frac{yv_*}{A_+\nu}\right)\right]\frac{\delta}{4}, \tag{3}$$

where $\kappa = 0.4$, $A_+ = 26$.

Let us examine the expression for the longitudinal velocity fluctuations in the boundary layer that Prandtl proposed

$$|\overline{u'}|=l\frac{\partial\bar{u}}{\partial y} \tag{4}$$

in greater detail, where ℓ is the mixing path length that differs in magnitude from the scale L used to define the turbulent viscosity. Experimental data indicate that $\overline{u'v'}$ and $\overline{u'^2}$ are interrelated nonlinearly. Then the nonlinear relationship between ℓ and L follows from a comparison of the formulas

$$-\overline{u'v'}=L^2\left(\frac{\partial\bar{u}}{\partial y}\right)^2;\quad \overline{u'^2}=l^2\left(\frac{\partial\bar{u}}{\partial y}\right)^2 \tag{5}$$

On the basis of a generalization of the experimental data [1, 3, 4], we propose an algebraic expression for the mixing path which yields an approximate description of the fluctuation profile in a viscous sublayer and the logarithmic domain of the boundary layer:

$$l=\begin{cases}0.8y, & y<0{,}156\,\mathrm{Re}^{0.13}\delta^{0.3},\\ 0{,}062\,\mathrm{Re}^{0.13}(y/\delta)^{0.7}\delta, & y\geqslant 0{,}156\,\mathrm{Re}^{0.13}\delta^{0.3}.\end{cases} \tag{6}$$

For a more exact determination of the fluctuations in the boundary layer, we propose to use a model based on the balance equation for the square of the longitudinal velocity fluctuations. This latter is derived from the Navier–Stokes equations by using the standard Friedman–Keller operation [7] and can be written in the boundary-layer approximation in the form

$$\rho\bar{u}\frac{\partial\overline{u'^2}}{\partial x}+\rho\bar{v}\frac{\partial\overline{u'^2}}{\partial y}=\frac{\partial}{\partial y}\left(\rho\nu\frac{\partial\overline{u'^2}}{\partial y}-\rho\,\overline{v'u'^2}-2\overline{v'p'}\right)-2\rho\overline{u'v'}\frac{\partial\bar{u}}{\partial y}-2\rho\nu\overline{\frac{\partial u'}{\partial y}\frac{\partial u'}{\partial y}}. \tag{7}$$

It is known that in the logarithmic boundary-layer domain the generation of the kinetic energy of turbulence (meaning also the longitudinal velocity fluctuations that introduce the greatest contribution) is equal to its dissipation [7]. If it is assumed, in addition, that the Prandtl hypothesis is valid, then the dissipative term can be approximated by the extensively utilized expression

$$2\rho\nu\overline{\frac{\partial u'}{\partial y}\frac{\partial u'}{\partial y}}=2\rho A(\nu+\alpha\nu_T)\frac{\overline{u'^2}}{l^2},$$

where A and α are certain constants.

If the coefficient of turbulent diffusion of the square of the longitudinal velocity fluctuations is considered proportional to ν_T, then by taking account of the molecular diffusion and dissipation, (7) takes the form

$$\rho\bar{u}\frac{\partial\overline{u'^2}}{\partial x}+\rho\bar{v}\frac{\partial\overline{u'^2}}{\partial y}=\frac{\partial}{\partial y}\left[(\nu+\gamma\nu_T)\frac{\partial\overline{u'^2}}{\partial y}\right]+2\rho\nu_T\left(\frac{\partial\bar{u}}{\partial y}\right)^2-2A\rho(\nu+\alpha\nu_T)\frac{\overline{u'^2}}{l^2}, \tag{8}$$

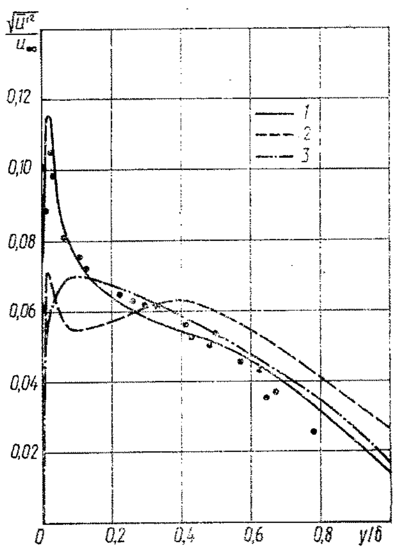

Fig. 1

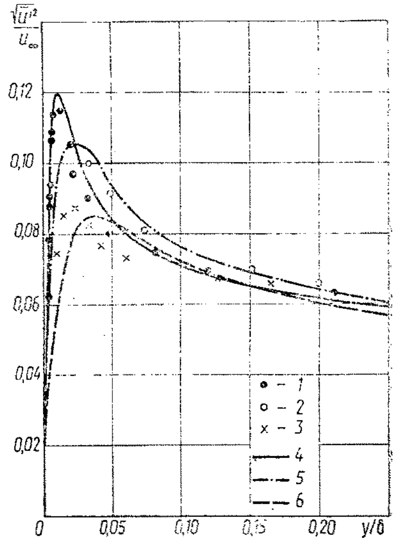

Fig. 2

Fig. 1. Analysis of velocity fluctuations on a plate according to (7) with different values of the scale of turbulence (Re $4.2 \cdot 10^6$): 1) ℓ from (5); 2) ℓ = 0.4y; 3) ℓ from (3); the points are from experiment [1].

Fig. 2. Analysis of fluctuations in the near-wall domain for different Re: 1) experiment [1]; 2) experiment [3]; 3) experiment [4]; lines 4-6 from computation: 4) Re = $4.2 \cdot 10^6$; 5) $1.2 \cdot 10^6$; 6) $5 \cdot 10^5$.

where A, α, and γ are constants, and ν_T is determined from (2) and (3). The boundary condition is that the square of the velocity fluctuation be zero at the wall and in the external flow.

Such mutual velocity and fluctuation profiles in the boundary layer permitted solution of the problem by using a self-similar variable. Equations (1) and (7) were reduced to locally self-similar form and solved numerically in the Reynolds number range Re = 10^5-10^7 with different values of the scale of turbulence in the dissipative term.

The results of computing the average velocities were compared with the experimental data presented in [8]. The difference between the computation and experiment did not exceed 5%. The best results on the fluctuation profile were obtained by using the scale defined by (6). Application of the scale in the form L = κy, as well as the scale (3) and the like, such as utilized in [9], did not yield a satisfactory description of the fluctuation profile (Fig. 1).

Computations of the fluctuations by using a differential model are in considerably better agreement with the experimental data than for the computation using (4) and (6), and afford the possibility of describing the behavior of the fluctuations near a wall where they reach their maximal value (Fig. 2). The computations displayed an increase in the maximum fluctuation as the number Re grows, as is confirmed by certain experimental data as well as by computations of the kinetic energy of turbulence [6]. The growth in the fluctuation peak can be explained by a rise in the velocity gradient near the wall as the number Re grows, which specifies an increase in generation of the fluctuations.

It is known that the maximum fluctuation is observed on the viscous sublayer boundary, more accurately in the buffer zone. The approximation of the maximum to the wall obtained in the computations in the coordinates y/δ corresponds to a diminution in the viscous sublayer thickness relative to the total boundary-layer thickness as the number Re grows.

Since equations analogous to (8) are used in turbulent flow computations, the influence of the constant therein on the nature of the solution was investigated. As is disclosed, the coefficients in the dissipative term exert the greatest influence on the solution. An increase in the complex Aα results in a diminution in the fluctuation amplitude, where the solutions obtained for different Aα are almost similar (Fig. 3a). A change in the coefficient A

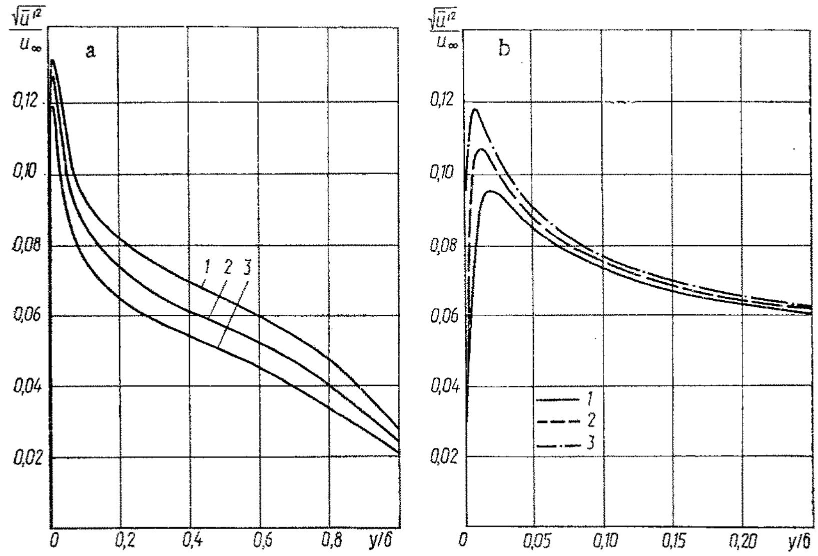

Fig. 3. Influence of values of the constants in the dissipative term on the solution of (7): a) 1) $\alpha A = 0.5$; 2) 1.0; 3) 2; b) 1) $\alpha = 1.0$; 2) 0.5; 3) 0.

for a constant $A\alpha$ exerts noticeable influence on the magnitude of the maximum. The growth of A (which corresponds to an increase in the viscous dissipation of the fluctuations) diminishes the height of the peak, without exerting influence on the remainder of the solution in practice (Fig. 3b). A change in the coefficient γ in the diffusion term within the limits $\gamma = 0.25$-4.0 affects the solution of (8) slightly.

The best agreement with experimental data is obtained for the following values of the elements $\gamma = 1.0$, $\alpha = 0.8$, $A = 1.25$.

NOTATION

x, y, longitudinal and transverse coordinates; $\bar{u}$, $\bar{v}$, average longitudinal and transverse velocity components; u', v', longitudinal and transverse velocity fluctuation components; ρ, fluid density; ν, coefficient of kinematic viscosity; ν_T, kinematic velocity coefficient in the case of turbulent viscosity; L, mixing path used to calculate the turbulent viscosity; ℓ, mixing path relating the magnitude of the velocity fluctuation to the gradient of the average velocity; κ, Karman constant; δ, boundary-layer thickness; Re, Reynolds criterion; $v_* = \sqrt{\tau_w/\rho}$, dynamic velocity; τ_w, friction stress on the wall; A_+, constant in the Van Driest formula for the damping factor; A, α, γ, constants in the balance equation for the square of the longitudinal component of the velocity vector.

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STABILITY OF A LOW-TEMPERATURE HELIUM FLOW IN HEATED CHANNELS

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UDC 621.039.534.36

The stability of a turbulent helium flow under forced convection is investigated in channels with different hydraulic characteristics.

The stability of a helium flow in heated channels must be investigated to determine the stable cooling conditions for different cryoenergetic apparatus in the 4-10°K temperature range at pressures up to 1.5 MPa.

The spontaneous origination of wall pressure and temperature fluctuations was observed in fluid heating for sub- and supercritical pressures in many experimental papers [1-3], say.

Among the different kinds of vibrational processes in fluids, the so-called density wave fluctuations are most widespread. Characteristic for them is the propagation of density, enthalpy, and mass-flow perturbations along a channel at the fluid flow velocity, which can damp out or grow with time for a definite relationship between the mass-flow and the thermal load.

The results of a number of theoretical and experimental investigations of such fluctuations in cryogenic systems cooled by supercritical-pressure helium have recently been published. The Nyquist frequency criterion based on the principle of an argument was used in [4] to analyze stability, and permitted estimation of a number of versions of cooling with the thermodynamic properties of the gas taken into account.

On the basis of a simplified model using an approximate description of the thermodynamic properties of helium in the near-critical region, an equation was obtained in [5] for the stability boundary in two dimensionless parameters $\psi_d = \Delta P_1/\Delta P_2$ is the ratio of the pressure drops in the input and output chokes, and $R = v_2/v_1$ is the degree of gas expansion. The pressure over the channel length was assumed constant in the computations.

The boundary of the beginning of the origination of vibrations by using analogous criteria was constructed earlier on the basis of experimental investigations [1]: the relative hydraulic drag

$$\psi = \frac{\Delta P_1 + \Delta P_{1\text{K}}}{\Delta P_{2\text{K}} + \Delta P_2} \tag{1}$$

and the degree of expansion $R = (v_2 - v_1)/v_1$. Here, ΔP_1 is the pressure drop at the input throttle; ΔP_{1K} is the channel hydraulic drag between the input throttle and the section with the pseudocritical temperature T_m of the flow; ΔP_{2K} is the channel hydraulic drag between the section with temperature T_m and the output throttle, and ΔP_2 is the hydraulic drag of the output throttle.

A disagreement between the results of a computation [4] and the stability boundary obtained on the basis of experiments was noted in [1]. A discrepancy is also noticeable in comparing the results of computations [5] and test data, especially in the domain of small values of ψ.

G. M. Krzhizhanovskii Energy Institute, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 49, No. 5, pp. 727-733, November, 1985. Original article submitted June 29, 1984.

 0022-0841/85/4905-1262$09.50